

Spontaneous currents in a superconductor with $s+is$ symmetry

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We analyze $s + is$ state proposed as a candidate superconducting state for strongly hole-doped $\text{Ba}_{1-x}\text{K}_x\text{Fe}_2\text{As}_2$. Such a state breaks time-reversal symmetry (TRS) but does not break any other discrete symmetry. We address the issue whether TRS breaking alone can generate spontaneous currents near impurity sites, which could be detected in, e.g., μSR experiments. We argue that there are no spontaneous currents if only TRS is broken. However, supercurrents do emerge if the system is put under external strain and C_4 lattice rotation symmetry is externally broken.

Introduction The search for a truly unconventional superconductivity which breaks time-reversal symmetry (TRS) in addition to $U(1)$ overall phase symmetry of a superconducting order parameter continues to attract a lot of attention in the physics community.^{1–3} Systems which break TRS exhibit a wealth of fascinating properties and are highly sought after for applications^{4–14}. The TRS breaking pairing states have been proposed in the two-dimensional ^3He (Ref. 2) and for the fractional quantum Hall effect at $5/2$ filling.^{15,16} In solid-state realizations, TRS-breaking (TRSB) $p_x + ip_y$ superconductivity has been proposed for Sr_2RuO_4 (Ref. 17) and found to be in agreement with the measurements of the polar Kerr effect, specifically designed to measure TRS breaking.¹⁸ There are no experimental realizations yet of TRSB spin-singlet superconductivity, although $d_{x^2-y^2} + id_{xy}$ state has been proposed theoretically for hexagonal systems near van-Hove doping, including doped graphene,^{19–21} SrPtAs ²² and, possibly, cobaltates.²³

The search for TRS breaking superconductivity intensified with the discovery of Fe-based superconductors (FeSCs). These systems have multiple Fermi surfaces, and intra-pocket and inter-pocket interactions vary along each Fermi surface as the consequence of different orbital compositions of low-energy excitations.²⁴ This variation opens a possibility that the pairing interaction is attractive in more than one channel. In FeSCs, the two leading candidates are s^{+-} and $d_{x^2-y^2}$ channels. The majority of researchers believe that in moderately doped FeSCs s^{+-} superconductivity wins, but some experiments on heavily hole-doped system $\text{Ba}_{1-x}\text{K}_x\text{Fe}_2\text{As}_2$ with $x = 1$ were interpreted in favor of $d_{x^2-y^2}$ superconductivity²⁵. This interpretation is not universally accepted²⁶, but if it is correct, then one can expect that there will be a mixed state at $x \leq 1$, in which both s and d components are present. According to calculations²⁷ relative phase between s and d -components is $\pm\pi/2$ (an $s + id$ state). In such a state time-reversal (TR) and C_4 lattice rotational symmetry are simultaneously broken, together with $U(1)$ phase symmetry, and this gives rise to a number of non-trivial properties, including circulating supercurrents near a nonmagnetic impurity.²⁸

In a separate line of research, several groups argued that, even if the gap symmetry in $\text{Ba}_{1-x}\text{K}_x\text{Fe}_2\text{As}_2$ remains s -wave for all x , the multi-pocket nature of FeSCs still allows for non-trivial superconductivity at intermediate $x \leq 1$, which break TRS. The argument is that the s^{+-} order parameter (the gap) at $x = 1$ (KFe_2As_2) is qualitatively different from the one at optimal doping ($x \approx 0.4$), where both hole and electron pockets are present. At optimal doping the most natural s^{+-} state is the one in which the gaps on all hole pockets have the same sign, opposite to that on electron pockets. At $x = 1$ (i.e., in KFe_2As_2) only hole pockets are present²⁹ and, if the gap remains s -wave, it must change sign between the two inner hole pockets.³⁰ It has been argued³¹ that the transformation of one s -wave structure into the other is not continuous at low enough T and involves an intermediate state in which the phases of the gaps on the two inner hole pockets differ by $0 < \alpha < \pi$ (see Fig. 2). Such a state breaks TRS, despite that it has pure s -wave symmetry, and was termed $s + is$.

The issue we discuss in this letter is how to detect the $s + is$ state experimentally. One option is to detect low-energy Leggett-type modes in the $s + is$ state^{31,32} by, e.g., Raman measurements. But it would be much more desirable to have experimental probes which would directly detect TRS breaking. In this respect, $s + is$ state presents a challenge. The TRS-broken states studied before break other discrete symmetries in addition TR, e.g., $s + id_{x^2-y^2}$ state breaks C_4 symmetry and $d_{x^2-y^2} + id_{xy}$ breaks mirror symmetries. The $s + is$ state only breaks TRS but keeps lattice symmetries intact. We show that in this situation there are no circulating supercurrents near a non-magnetic impurity. We show, however, that supercurrents do develop if a system with $s + is$ gap is put under external strain that breaks the C_4 lattice rotational symmetry. Our results are in agreement with recent zero-field muon-spin relaxation study of superconducting $\text{Ba}_{1-x}\text{K}_x\text{Fe}_2\text{As}_2$ for $0.5 < x < 0.9$ (Ref. [33]). The measurements on polycrystalline samples, which do not exhibit the mesoscopic phase separation, showed no evidence of spontaneous internal magnetic fields at temperatures down to 0.02 K. We propose to preform the

same μSR measurement under external strain.

s + id superconductor To set the stage for our analysis of $s + is$ state it is instructive to consider first $s + id$ state for which numerical calculations²⁸ have shown the presence of spontaneous currents around an inhomogeneity. This will help us understand the distinction between $s + id$ and $s + is$ states and set up necessary conditions for the existence of currents in an $s + is$ superconductor. The free energy for a candidate $s + id$ system can be written as the combination of the homogeneous and spatially varying parts: $\mathcal{F} = \mathcal{F}_h + \mathcal{F}_s$, where

$$\begin{aligned}\mathcal{F}_h &= \alpha_s |\Delta_s|^2 + \alpha_d |\Delta_d|^2 + \beta_1 |\Delta_s|^4 + \beta_2 |\Delta_d|^4 \\ &\quad + \beta_3 |\Delta_s|^2 |\Delta_d|^2 + \beta_4 (\Delta_s^* \Delta_s^* \Delta_d \Delta_d + \text{c.c.}) \\ \mathcal{F}_s &= \gamma_s |\vec{D} \Delta_s|^2 + \gamma_d |\vec{D} \Delta_d|^2 \\ &\quad + \gamma_{sd} \left[(\vec{D}_x \Delta_s)^* \vec{D}_x \Delta_d - (\vec{D}_y \Delta_s)^* \vec{D}_y \Delta_d + \text{c.c.} \right] \end{aligned} \quad (1)$$

Here Δ_d is the magnitude of $d_{x^2-y^2}$ gap ($= \Delta_d \cos 2\theta$) and the integration over θ is already carried out. The two order parameters are $U(1)$ fields $\Delta_s = \Delta e^{i\phi_s}$ and $\Delta_d = \Delta e^{i\phi_d}$. In the spatially varying part $\vec{D} \equiv -i\vec{\partial} - \frac{2e}{c}\vec{A}$ and all derivatives act of the center of mass co-ordinate of the Cooper pair.³⁴ We assume that the parameters α_i and β_i of the homogeneous part \mathcal{F}_h are such that the ground state is a TRSB $s + id$ superconductor with $\phi_d - \phi_s = \pm\pi/2$.

The term with the prefactor γ_{sd} depends on the relative phase $\phi_s - \phi_d$ of the two order parameters, but not on the cumulative phase $\phi_s + \phi_d$. This term is consistent with the symmetry of $s + id$ state as it remains invariant if one changes the relative phase $\phi_d - \phi_s$ by π and simultaneously rotates the reference frame by 90° . Both symmetry operations change $\Delta_d \rightarrow -\Delta_d$, and \mathcal{F} is invariant under the product of these two operations.

We now show that the γ_{sd} term in the free energy (1) gives rise to circulating currents around inhomogeneities. For this, we introduce an isotropic impurity at $\mathbf{r} = \mathbf{0}$, obtain coordinate-dependent $\Delta_s(\mathbf{r})$ and $\Delta_d(\mathbf{r})$, and compute the current density $\vec{j} = -\frac{\partial \mathcal{F}_s}{\partial \vec{A}}|_{\vec{A}=0}$. We follow Ref. 28 and assume that an impurity introduces coordinate dependencies of the prefactors α_i in (1) via $\alpha_i \rightarrow \alpha_i + \alpha_{\text{imp},i}(\vec{r})$ ($i = s, d$), where $\alpha_{\text{imp},i}(\vec{r})$ is a decreasing function of r . We take the same $\alpha_{\text{imp},i} = \alpha_{0,i} e^{-(r/r_{0,i})^2}$ as in Ref. 28, but the results would be qualitatively similar for any isotropic impurity potential. To simplify the presentation, we assume $\alpha_s = \alpha_d$ and drop the index i .

We treat $\alpha_{\text{imp}}(\vec{r})$ as a small perturbation and linearize the response around the TRSB state by setting $\phi_s = 0$ and $\phi_d = \pi/2$ away from impurity and expanding Δ_s and Δ_d as $\Delta_s = \Delta(1 + m_s + i\phi_s)$ and $\Delta_d = i\Delta(1 + m_d + i\phi_d)$, where $m_s, m_d, \phi_s, \phi_d = \mathcal{O}(\alpha_0)$ are all functions of \vec{r} . Minimizing \mathcal{F} with respect to variations of Δ_s and Δ_d we obtain³⁴

$$\begin{aligned}\gamma_d m_d &= \gamma_s m_s = -\frac{1}{2}(t_+ + t_-), \\ \sqrt{\gamma_s \gamma_d} \phi_d &= -\sqrt{\gamma_s \gamma_d} \phi_s = \frac{1}{2}(t_+ - t_-), \end{aligned} \quad (2)$$

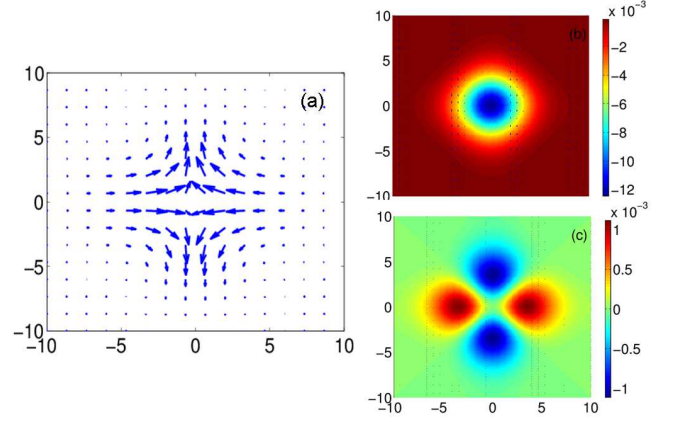


FIG. 1: (a) The profile of supercurrent (blue arrows) in real space in the $s + id$ superconductor around an isotropic impurity located at the origin. The current satisfies the continuity equation and creates a magnetic quadrupole moment centered on the impurity site. We set $\tilde{\gamma} = 1$, sample size $L = 10r_0$, $\delta = 0.5/r_0$. (b) and (c) are the amplitude and phase fluctuations, $\gamma_s m_s = \gamma_d m_d$ and $\sqrt{\gamma_s \gamma_d} \phi_d = -\sqrt{\gamma_s \gamma_d} \phi_s$, respectively. Note the d -wave symmetry of phase fluctuations.

where

$$t_{\pm} \equiv t_{\pm}(r) = \int \frac{d^2 k}{(2\pi)^2} \frac{e^{i\vec{k} \cdot \vec{r}} \tilde{\alpha}_{\text{imp}}}{k_x^2 (1 \pm \tilde{\gamma}) + k_y^2 (1 \mp \tilde{\gamma}) + \delta^2}. \quad (3)$$

In Eq. (3) $\tilde{\alpha}_{\text{imp}}$ is the Fourier transform of α_{imp} , $\tilde{\gamma} \equiv \gamma_{sd}/\sqrt{\gamma_s \gamma_d}$, and δ is the mass scale set by β_i in (1). The current density is expressed as

$$\vec{j} = -j_0 \left[\gamma_s \vec{\partial} \phi_s + \gamma_d \vec{\partial} \phi_d + \gamma_{sd} \vec{\partial}_d (m_d - m_s) \right], \quad (4)$$

where $\vec{\partial}_d = \hat{x} \partial_x - \hat{y} \partial_y$. Substituting the results for m_s, m_d, ϕ_s, ϕ_d from (2) we obtain

$$\vec{j} = \frac{j_0}{2} \frac{\gamma_s - \gamma_d}{\sqrt{\gamma_s \gamma_d}} \left[\vec{\partial}(t_+ - t_-) + \tilde{\gamma} \vec{\partial}_d(t_+ + t_-) \right]. \quad (5)$$

One can easily verify that \vec{j} satisfies the continuity equation $\vec{\nabla} \cdot \vec{j} = 0$. The difference $t_+ - t_-$ contains $\tilde{\gamma} \propto \gamma_{sd}$ as the overall factor (see Eq. (3), hence both terms in the square brackets in (29) scale with γ_{sd} , and $|\vec{j}| \propto \gamma_{sd}$. We see that the current is entirely due to the mixed γ_{sd} term in (1), which, we remind, remains invariant under the product of C_4 rotation and TR. The presence of two symmetry transformations, each of which changes Δ_d to $-\Delta_d$, is therefore crucial to obtain a non-zero circulating current around an impurity. The current $\mathbf{j}(\mathbf{r})$ is plotted in Fig. 1 and agrees well with the numerical results.²⁸

s + is superconductor We now apply the same phenomenological description to $s + is$ superconductor with order parameters Δ_s and $\Delta_{s'}$. The generic form of \mathcal{F} remains the same as in Eq. (1), we only need to replace Δ_d by $\Delta_{s'}$, γ_d by $\gamma_{s'}$, γ_{sd} by $\gamma_{ss'}$, and change the sign

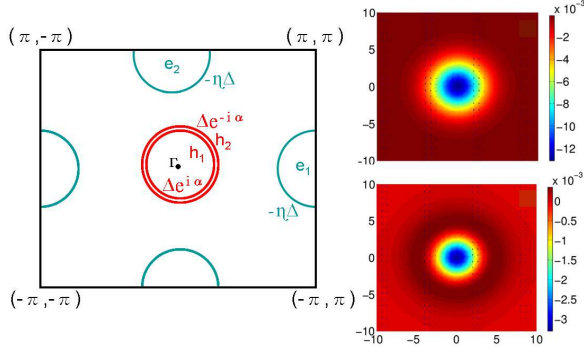


FIG. 2: Left: The Fermi surface of the microscopic model considered in the text. It has two hole pockets around the Γ point and two electronic pockets at $(\pi, 0)$ and $(0, \pi)$ points of the Brillouin zone. We tune the parameters of this model to the TRSB state and expand the gaps in spatial fluctuations around the impurity site in TRSB $s + is$ state to obtain the currents. Right: The spatial variation of the amplitude (m_1 , top) and the phase (ϕ_1 , bottom) of the gap on one of hole pockets without external strain. Observe that there is no phase modulation unlike in $s + id$ state.

between D_x and D_y parts of the mixed term, which for C_4 -preserving $s + is$ superconductor becomes

$$\gamma_{ss'} \left[(\vec{D}_x \Delta_s)^* \vec{D}_x \Delta_{s'} + (\vec{D}_y \Delta_s)^* \vec{D}_y \Delta_{s'} + \text{c.c.} \right], \quad (6)$$

Such term is allowed by symmetry as it again depends only on the relative phase of Δ_s and $\Delta_{s'}$ and is invariant under gauge transformation and TR operation. However, in distinction to $s + id$ case, there is no possibility to compensate the change of the phase of Δ'_s by π by a rotation of the coordinate frame by 90° . As a result, the contributions from this term must vanish in the $s + is$ state, hence it should not give rise to a circulating current. To see this explicitly, we calculate the \vec{j} using the cross-term with the derivatives in the form of Eq. (6). The current is still given by Eq. (2), but now $t_\pm = \alpha_{\text{imp}} / (k^2 + \delta^2)(1 \pm \tilde{\gamma})$. Substituting this into (2) we find that \vec{j} vanishes, as expected:

$$\vec{j} = \frac{j_0}{2} \frac{\gamma_s - \gamma_{s'}}{\sqrt{\gamma_s \gamma_{s'}}} \vec{\partial} \frac{\alpha_{\text{imp}}}{k^2 + \delta^2} \left(\frac{-2\tilde{\gamma}}{1 - \tilde{\gamma}^2} + \frac{2\tilde{\gamma}}{1 - \tilde{\gamma}^2} \right) = 0. \quad (7)$$

We now substantiate this reasoning with the analysis of the minimal band-resolved model which displays $s + is$ superconductivity. The model consists of two Γ -centered hole pockets and two electron pockets centered around $(0, \pi)$ and $(\pi, 0)$ in the one-Fe Brillouin zone, as shown in Fig. 2. (In the physical two-Fe Brillouin zone, these two electron pockets are centered at the same (π, π) point and hybridize into inner and outer pockets.) We label the gap functions on hole pockets as $\Delta_{h_1}, \Delta_{h_2}$ and use Δ_{e_1} and Δ_{e_2} for inner and outer electron pockets. As discussed in Ref. 31, an $s + is$ can be realized at large hole doping where $\Delta_{h_1} = \Delta_1 e^{i\alpha_1}$, $\Delta_{h_2} = \Delta_2 e^{i\alpha_2}$, and $\alpha_1 - \alpha_2$ is a fraction of π . The spatially varying part of

the free energy for this state is

$$\begin{aligned} \mathcal{F}_s = & a_1 |\vec{D} \Delta_{h_1}|^2 + a_2 |\vec{D} \Delta_{h_2}|^2 + a_3 |\vec{D} \Delta_{e_1}|^2 + \\ & a_4 |\vec{D} \Delta_{e_2}|^2 + a_h \left[(\vec{D} \Delta_{h_1})^* \cdot (\vec{D} \Delta_{h_2}) + \text{c.c.} \right] \\ & + a_{h_1 e_1} \left[(\vec{D} \Delta_{e_1})^* \cdot (\vec{D} \Delta_{h_1}) + \text{c.c.} \right] + [e_1 \rightarrow e_2] \\ & + a_{h_2 e_1} \left[(\vec{D} \Delta_{e_1})^* \cdot (\vec{D} \Delta_{h_2}) + \text{c.c.} \right] + [e_1 \rightarrow e_2] \\ & + a_e \left[(\vec{D} \Delta_{e_1})^* \cdot (\vec{D} \Delta_{e_2}) + \text{c.c.} \right] \end{aligned} \quad (8)$$

The terms with products $(\vec{D} \Delta_i)^* \cdot (\vec{D} \Delta_j)$ with $i \neq j$ depend on the relative phases of Δ_i and Δ_j and their structure reproduces that in Eq. (6).

Differentiating the free energy with respect to vector potential we obtain the current $\vec{j} = -\frac{\partial \mathcal{F}_s}{\partial \vec{A}}|_{\vec{A}=0}$ in the form

$$\begin{aligned} i \frac{j_x}{2e/c} = & a_1 \Delta_{h_1}^* \vec{\partial}_x \Delta_{h_1} + a_2 \Delta_{h_2}^* \vec{\partial}_x \Delta_{h_2} + a_3 \Delta_{e_1}^* \vec{\partial}_x \Delta_{e_1} \\ & + a_4 \Delta_{e_2}^* \vec{\partial}_x \Delta_{e_2} + a_h [\Delta_{h_1}^* \vec{\partial}_x \Delta_{h_2} - \text{c.c.}] \\ & + a_{h_1 e_1} [\Delta_{e_1}^* \vec{\partial}_x \Delta_{h_1} - \text{c.c.}] + [e_1 \rightarrow e_2] \\ & + a_{h_2 e_1} [\Delta_{e_1}^* \vec{\partial}_x \Delta_{h_2} - \text{c.c.}] + [e_1 \rightarrow e_2] \\ & + a_e [\Delta_{e_1}^* \vec{\partial}_x \Delta_{e_2} - \text{c.c.}], \end{aligned} \quad (9)$$

where $g\vec{\partial}h \equiv g\partial h - h\partial g$. The expression for j_y is obtained by interchanging $\vec{\partial}_x$ by $\vec{\partial}_y$.

We perform the same computational steps as for $s + id$ superconductor: introduce an impurity at $\vec{r} = 0$ and expand the order parameters to linear order around the homogeneous solution for $s + is$ state. We first consider the simplest case when we treat the two hole pockets and the two electron pockets as identical in the spatially varying part of the free energy, i.e., set $a_1 = a_2, a_3 = a_4, a_{h_i e_j} = a_{he}$ in Eq. (8). The homogeneous solution for identical hole and identical electron pockets is³¹ $\Delta_{h_1} = \Delta e^{i\alpha}$, $\Delta_{h_2} = \Delta e^{-i\alpha}$, $\Delta_e = -\eta \Delta$, where η and α are functions of doping. For the critical doping where T_c^{TRSB} is the largest, $\eta = \sqrt{2}$ and $\alpha = \pi/4$. We expand the gaps as $\Delta_{h_1} = \Delta e^{i\alpha}(1 + m_1 + i\phi_1)$, $\Delta_{h_2} = \Delta e^{-i\alpha}(1 + m_2 + i\phi_2)$, $\Delta_{e_1} = \Delta_{e_2} = -\eta \Delta(1 + m_e + i\phi_e)$, substitute the expansion into (9) and obtain for the current

$$\begin{aligned} j_x = & j_0 [(a_1 + a_h \cos 2\alpha - 2a_{he} r \cos \alpha) \partial_x (\phi_1 + \phi_2) \\ & + (2a_3 r^2 + 2a_e r^2 - 4a_{he} r \cos \alpha) \partial_x \phi_e \\ & + (a_h \sin 2\alpha - 2a_{he} r \sin \alpha) \partial_x (m_1 - m_2)], \end{aligned} \quad (10)$$

and j_y is obtained by interchanging x and y . Minimizing \mathcal{F}_s with respect to variations m_i, ϕ_i, m_e and ϕ_e we obtain that $\phi_e = 0$ and $m_1 = m_2, \phi_1 = -\phi_2$, where

$$m_1 + i\phi_1 = \int \frac{d^2 k}{(2\pi)^2} e^{i\vec{k} \cdot \vec{r}} \frac{a_3 + a_e + 2a_{he}}{G} e^{i\alpha} \frac{-\tilde{\alpha}_{\text{imp}}}{k^2 + \delta^2},$$

and $G \equiv (a_1 + a_h)(a_3 + a_e) - 4a_{he}^2 \eta$. Substituting this into Eq. (10) we see that each term in (10) vanishes, i.e.,

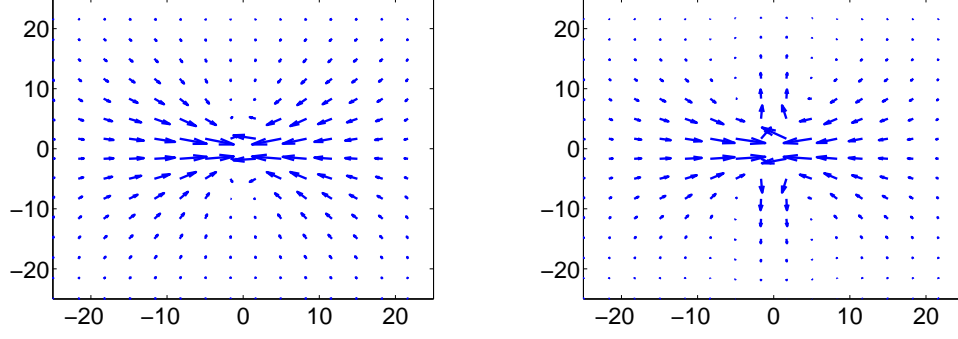


FIG. 3: The profile of a supercurrent for the $s+is$ state. We set the parameter ϵ , related to external breaking of C_4 symmetry, to be $\epsilon = 0.1\sqrt{a_1 a_2}$, and $0.3\sqrt{a_1 a_2}$ in left and right panels, respectively. We also set $a_h = 0.5\sqrt{a_1 a_2}$, $\delta = 0.1/r_0$, and the sample size $L = 25r_0$ where, we remind, r_0 is related to impurity-induced correction to the prefactor α in the quadratic part in the free energy via $\alpha_{\text{imp}}(\vec{r}) \propto e^{-(r/r_0)^2}$. The asymmetry in the current pattern from x to y is due to the applied strain.

$\vec{j} = 0$. We extended the analysis to non-equivalent hole pockets and to distinct inner and outer electron pockets, and obtained the same result³⁴: spontaneous supercurrent around an impurity vanishes. In these cases, individual contributions to the current do not vanish but the variations of the gap magnitudes ($\partial_x m$ -terms) cancel the variations of the phases ($\partial_x \phi$ terms).

$s+is$ superconductor under external strain The comparative analysis of $s+id$ and $s+is$ cases shows a way how one can generate supercurrents in an $s+is$ superconductor — one has to externally break C_4 rotational symmetry. This can be achieved, for example, by application of a small uniaxial strain. For Eq. (6) breaking of C_4 implies that $\gamma_{s,s'}$ along x and y directions become non-equivalent: $\gamma_{ss}^x = \gamma_{ss} + \gamma_{ss}^*$ and $\gamma_{ss}^y = \gamma_{ss} - \gamma_{ss}^*$. Then the cross-term becomes the sum of (6) and the new term

$$\gamma_{ss'}^* \left[(\vec{D}_x \Delta_s)^* \vec{D}_x \Delta_{s'} - (\vec{D}_y \Delta_s)^* \vec{D}_y \Delta_{s'} + \text{c.c.} \right], \quad (11)$$

This last term has the same form as for $s+id$ superconductor and should give rise to a supercurrent. Another way to state the same is to recall that breaking of C_4 mixes s -wave and d -wave channels, i.e. s -wave gaps change from Δ_a to $\Delta_{a,1} + \epsilon \Delta_{a,2} \cos 2\theta_a$, where $a = s, s'$. Selecting the products $\Delta_{s,1} \Delta_{s',2}$ and $\Delta_{s',1} \Delta_{s,2}$, we recover Eq. (11) with $\gamma_{ss'}^* \propto \epsilon$. Note in passing that in KFe_2As_2 ϵ and $\gamma_{ss'}^*$ are additionally enhanced because s -wave and $d_{x^2-y^2}$ channels are nearly degenerate^{27,30}.

We now apply this reasoning to our model for $s+is$ superconductor. To simplify the presentation, we assume that the effect of strain is the strongest on the a_h term in \mathcal{F}_s , set $a_h^x = a_h + \epsilon$ and $a_h^y = a_h - \epsilon$, and ignore $a_{h_i e_j}$ and a_e terms which involve electron pockets. Within this approximation, the currents are given by

$$j_x = j_0 [\partial_x (a_1 \phi_1 + a_2 \phi_2) + (a_h + \epsilon) \cos 2\alpha \partial_x (\phi_1 + \phi_2) + (a_h - \epsilon) \sin 2\alpha \partial_x (m_1 - m_2)], \quad (12)$$

and j_y is obtained by interchanging $x \rightarrow y$ and $\epsilon \rightarrow -\epsilon$.

The coordinate-dependent functions m_j and ϕ_j ($j = 1, 2$) are obtained by minimizing \mathcal{F}_s with respect to fluctuations. Performing the same calculations as before we obtained

$$j_x = 2j_0 \epsilon \sin \alpha \frac{a_1 - a_2}{\sqrt{a_1 a_2}} \partial_x \int \frac{d^2 k}{(2\pi)^2} e^{i\vec{k} \cdot \vec{r}} \left(\frac{\tilde{\alpha}_{\text{imp}} k_y^2}{Z} \right),$$

$$j_y = -2j_0 \epsilon \sin \alpha \frac{a_1 - a_2}{\sqrt{a_1 a_2}} \partial_y \int \frac{d^2 k}{(2\pi)^2} e^{i\vec{k} \cdot \vec{r}} \left(\frac{\tilde{\alpha}_{\text{imp}} k_x^2}{Z} \right), \quad (13)$$

where $Z = a_1 a_2 k^4 - [a_h k^2 + \epsilon(k_x^2 - k_y^2)]^2$. We see that the current is non-zero only when $\alpha \neq 0, \pm\pi$, $\epsilon \neq 0$ (i.e. when TRS and C_4 symmetry are simultaneously broken) and $a_1 \neq a_2$ (i.e. when the two hole pockets have inequivalent fluctuations (this is analogous to the condition $\gamma_s \neq \gamma_d$ for $s+id$ superconductor; see Eq. (29)). We also emphasize that, although electron pockets do not contribute to \vec{j} in Eq. (13), they are essential to have $\alpha \neq 0, \pm\pi$. Note also that the continuity equation $\vec{\nabla} \cdot \vec{j} = 0$ is satisfied.

We show the current profile for \vec{j} given by Eq. (13) in Fig. It is quite similar to the one for $s+id$ superconductor. Superconductivity with $s+is$ symmetry has been proposed for KFe_2As_2 and we call for μSR measurements in this material under external strain.

Conclusions In this paper we analyzed $s+is$ state proposed as a candidate superconducting state for strongly hole-doped $\text{Ba}_{1-x}\text{K}_x\text{Fe}_2\text{As}_2$. This state breaks TRS but does not break any other discrete symmetry. We addressed the issue whether TRS breaking alone can generate spontaneous supercurrents, which can potentially be detected in μSR experiments. Our conclusion is that there are no currents if only TRS is broken. We argue, however, that spontaneous supercurrents do emerge in an $s+is$ superconductor if the system is put under external strain and C_4 lattice rotation symmetry is externally broken.

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Supplementary Material

A. Free energy analysis and spontaneous supercurrents in $s + id$ state

We discuss here in more detail the properties of the Free energy \mathcal{F} presented in Eq. (1) of the main text (MT) and derive the expression for the supercurrent – Eq. (5) of the MT. We first discuss the properties of the homogenous part of the Free energy given by:

$$\mathcal{F}_h = \alpha_s |\Delta_s|^2 + \alpha_d |\Delta_d|^2 + \beta_1 |\Delta_s|^4 + \beta_2 |\Delta_d|^4 + \beta_3 |\Delta_s|^2 |\Delta_d|^2 + \beta_4 (\Delta_s^* \Delta_s^* \Delta_d \Delta_d + \text{c.c.}) \quad (14)$$

We set the coefficients of the quadratic term to be the same because we want to tune to the point where the time-reversal symmetry breaking (TRSB) state starts right at T_c . If α_d and α_s differ a bit, TRSB state still emerges, but at $T < T_c$.

Some properties of \mathcal{F} at $\alpha_d = \alpha_s = \alpha$ are

- (1) TRSB is realized only when $\beta_4 > 0$.
- (2) $|\Delta_s|^2 / |\Delta_d|^2 = (2\beta_2 + 2\beta_4 - \beta_3) / (2\beta_1 + 2\beta_4 - \beta_3)$.
- (3) The phases of the order parameters differ by $\pm\pi/2$. We will set the cumulative phase such that $\phi_s = 0$ and $\phi_d = \pm\pi/2$. For the sake of convenience we choose $\beta_1 = \beta_2$ in which case $|\Delta_s| = |\Delta_d|$.

If there is an impurity at the origin, there will be a spatially varying part of the Free energy given by:

$$\mathcal{F}_s = \gamma_s |\vec{D} \Delta_s|^2 + \gamma_d |\vec{D} \Delta_d|^2 + \gamma_{sd} \left((\vec{D}_x \Delta_s)^* \vec{D}_x \Delta_d - (\vec{D}_y \Delta_s)^* \vec{D}_y \Delta_d + \text{c.c.} \right), \quad (15)$$

where $\vec{D} \equiv -i\vec{\partial} - \frac{2e}{c}\vec{A}$. We follow Ref. [28] and model the effect of impurity by replacing α_2 by $\alpha_2 + \alpha_{\text{imp}}(\vec{r})$, where $\alpha_{\text{imp}} = \alpha_0 e^{-(r/r_0)^2}$.

Minimizing \mathcal{F} leads to the coupled Ginzburg Landau (GL) equations-

$$\alpha_2 \Delta_s + 2\beta_1 |\Delta_s|^2 \Delta_s + \beta_3 |\Delta_d|^2 \Delta_s + \beta_4 \Delta_d^2 \Delta_s^* + \gamma_s (D_x^2 + D_y^2) \Delta_s + \gamma_{sd} (D_x^2 - D_y^2) \Delta_d = 0 \quad (16)$$

$$(s \leftrightarrow d). \quad (17)$$

The current is

$$\vec{j} = \vec{j}_s + \vec{j}_d + \vec{j}_{sd} \quad (18)$$

$$\vec{j}_s = -i \frac{2e}{c} \gamma_s \left[\Delta_s^* \vec{\partial} \Delta_s \right] + \text{c.c.} \quad (19)$$

$$\vec{j}_d = -i \frac{2e}{c} \gamma_d \left[\Delta_d^* \vec{\partial} \Delta_d \right] + \text{c.c.} \quad (20)$$

$$\vec{j}_{sd} = -i \frac{2e}{c} \gamma_{sd} \left[(\Delta_s^* \partial_x \Delta_d + \Delta_d^* \partial_x \Delta_s) \hat{x} - (x \rightarrow y) \right] + \text{c.c.} \quad (21)$$

The current \vec{j} needs to satisfy the continuity equation $\vec{\nabla} \cdot \vec{j} = 0$.

This non-linear system of partial differential equations can be solved numerically. However, to gain insight, we may choose to expand in the spatial variations induced by the impurity around the homogenous TRSB state and analyze a linearized version of the GL equations. As we shall see, this gives us an excellent intuitive picture of how the solutions for $\Delta_{s,d}(r)$ look like. We expand $\Delta_s = |\Delta_s| e^{i\phi_s}$ and $\Delta_d = |\Delta_d| e^{i\phi_d}$ near homogeneous TRSB state $\Delta_s = |\Delta|$, $\Delta_d = i|\Delta|$ as

$$\begin{aligned} \Delta_s &= \Delta + m_s + i\phi_s \Delta, \text{ and} \\ \Delta_d &= i(\Delta + m_d + i\phi_d \Delta). \end{aligned} \quad (22)$$

where the variations of the gap magnitudes(m_s and m_d) and the phases(ϕ_s and ϕ_d) are the spatially varying parts due to the presence of an impurity. Let us now rescale variables for convenience:

$$\alpha\Delta \rightarrow \alpha; \beta\Delta^3 \rightarrow \beta; \frac{m_s}{\Delta} \rightarrow m_s; \frac{m_d}{\Delta} \rightarrow m_d; \frac{r}{r_0} \rightarrow r; \gamma\Delta r_0^{-2} \rightarrow \gamma; \frac{4e\Delta r_0}{c} \rightarrow j_0. \quad (23)$$

If the impurity perturbation is weak, we can also neglect the feedback effect of \vec{A} and set it to zero in the calculations of m_i, ϕ_i ($i = s, d$). We will see that this does not lead to any inconsistency. Substituting Eq. (22) into the GL equations and expanding to linear order in m_i and ϕ_i we obtain

$$\begin{pmatrix} A - \gamma_s \partial^2 & B & 0 & \gamma_{sd} \partial_d^2 \\ B & A - \gamma_d \partial^2 & -\gamma_{sd} \partial_d^2 & 0 \\ 0 & -\gamma_{sd} \partial_d^2 & \tilde{A} - \gamma_s \partial^2 & -\tilde{B} \\ -\gamma_{sd} \partial_d^2 & 0 & \tilde{B} & -\tilde{A} + \gamma_d \partial^2 \end{pmatrix} \begin{pmatrix} m_s \\ m_d \\ \phi_s \\ \phi_d \end{pmatrix} = \begin{pmatrix} -\alpha_{\text{imp}} \\ -\alpha_{\text{imp}} \\ 0 \\ 0 \end{pmatrix}. \quad (24)$$

where $A = \alpha_2 + 6\beta_1 + \beta_3 - \beta_4$; $B = 2\beta_3 - 4\beta_4$; $\tilde{A} = \alpha_2 + 2\beta_1 + \beta_3 + \beta_4$; $\tilde{B} = 2\beta_4$; $\partial_d^2 = \partial_x^2 + \partial_y^2$; and $\partial_d^2 = \partial_x^2 - \partial_y^2$. In the same approximation, the current is given by

$$\vec{j} = -j_0 \left[\gamma_s \vec{\partial} \phi_s + \gamma_d \vec{\partial} \phi_d + \gamma_{sd} \vec{\partial}_d (m_d - m_s) \right], \quad (25)$$

where $\vec{\partial}_d = \partial_x \hat{x} - \partial_y \hat{y}$. To move on with analytic calculations we assume that $r_0 \ll l_0$, where $l_0 = \min \left\{ \sqrt{\gamma/\alpha}, \sqrt{\gamma/\beta\Delta^2} \right\}$. This inequality guarantees that $\gamma \gg A, B, \tilde{A}, \tilde{B}$. Then the system of GL equations decouples into two 2×2 sets. One is

$$\begin{pmatrix} -\gamma_d \partial^2 & -\gamma_{sd} \partial_d^2 \\ -\gamma_{sd} \partial_d^2 & -\gamma_s \partial^2 \end{pmatrix} \begin{pmatrix} m_d \\ \phi_s \end{pmatrix} = \begin{pmatrix} -\alpha_{\text{imp}} \\ 0 \end{pmatrix}, \quad (26)$$

and in the other set we replace $d \leftrightarrow s$; and ϕ_d by $-\phi_s$. This system of partial differential equations has a particular solution induced by α_{imp} and a general solution for $\alpha_{\text{imp}} = 0$. We can set the general solution to zero by arguing that in the absence of the impurity, the system has no spontaneous currents and since the impurity potential is short ranged, the particular solution, which gives the response to α_{imp} , can then be found using the Fourier Transform and matrix inversion. Performing the calculation we obtain (in the real space):

$$\begin{aligned} \gamma_d m_d = \gamma_s m_s &= -\frac{1}{2} (t_+ + t_-), \\ \sqrt{\gamma_s \gamma_d} \phi_d = -\sqrt{\gamma_s \gamma_d} \phi_s &= \frac{1}{2} (t_+ - t_-), \end{aligned} \quad (27)$$

where

$$t_{\pm} = \int \frac{d^2 k}{(2\pi)^2} e^{i\vec{k} \cdot \vec{r}} \frac{\tilde{\alpha}_{\text{imp}}}{k_x^2 (1 \pm \tilde{\gamma}) + k_y^2 (1 \mp \tilde{\gamma}) + \delta^2}, \quad (28)$$

where $\tilde{\gamma} \equiv \gamma_{sd}/\sqrt{\gamma_s \gamma_d}$, and $\tilde{\alpha}_{\text{imp}}$ is the fourier transform of α_{imp} . The parameter δ equals to zero in this calculation, but this is an artifact because we have ignored A, B, \dots relative to γ 's. If we keep A, B, \dots , we find that $\delta \neq 0$. We didn't compute δ explicitly and will use it just as a parameter which cuts off infrared singularity.

Substituting m_d, m_s, ϕ_d and $\phi_s = -\phi_d$ into the expression for the current we obtain

$$\vec{j} = \frac{j_0}{2} \frac{\gamma_s - \gamma_d}{\sqrt{\gamma_s \gamma_d}} \left[\vec{\partial} (t_+ - t_-) + \tilde{\gamma} \vec{\partial}_d (t_+ + t_-) \right]. \quad (29)$$

This is the result which we presented in the MT and plotted in Fig. 1 there. To check if this current satisfies the continuity equation, let us go back to momentum space and look at $\vec{k} \cdot \vec{j}$. We immediately find that $\vec{k} \cdot \vec{j} = -(k_x^2 + k_y^2)(t_+ - t_-) - \tilde{\gamma}(k_x^2 - k_y^2)(t_+ - t_-) = 0$. Like we said in the MT, our result is quite consistent with the numerical results for \vec{j} obtained in Ref. [28]. The message then is that the spontaneous current can be fully understood by studying the linearized GL equations and looking only at the spatially varying part of \mathcal{F} . We also emphasize that the existence of a non-zero \vec{j} is entirely due to the presence of γ_{sd} term in the Free energy. Indeed, if $\gamma_{sd} = 0$, then $\tilde{\gamma} = 0$ and $t_+ = t_-$, hence $\vec{j} = 0$.

We remind that, in this analysis, there is a characteristic length scale r_0 , that we expressed our lengths in. We repeat the assumptions that we made:

(1) $r_0 \gg k_F^{-1} \rightarrow$. This allows us to consider slow variations of our variables as functions of \vec{r} .

(2) $r_0 \ll l_0$. This justifies dropping $A, B, \tilde{A}, \tilde{B}$ compared to γ , i.e., we can neglect the homogeneous part of the Free energy in solving GL equations.

Combining the two approximations, we find that our analytic consideration is valid when $k_F^{-1} \ll r_0 \ll l_0$.

B. Test cases for the lack of supercurrents in $s + is$ state without external strain

Here we consider in detail the microscopic model discussed in the MT to show that there are no supercurrents induced in $s + is$ state. It was shown in the MT, that for the case when the spatially varying (fluctuating) part of the Free energy does not distinguish between the two hole pockets and between the two electron pockets, there are no supercurrents. Here we consider three more generic cases: (a) equivalent fluctuations of superconducting gaps on hole pockets and inequivalent gap fluctuations on the electron pockets; (b) inequivalent gap fluctuations on the hole pockets and equivalent gap fluctuations on the electron pockets; and (c) inequivalent gap fluctuations on the two hole pockets *and* on the two electron pockets.

We emphasize that in all these cases we expand around the homogeneous TRSB state with $\Delta_{h_1} = \Delta e^{i\alpha}, \Delta_{h_2} = \Delta e^{-i\alpha}$ and $\Delta_{e_1} = \Delta_{e_2} = -\eta\Delta$. The purpose to perform these checks is to show that, unless a spatial symmetry (in this case C_4) is broken, there cannot a supercurrent even if the various bands interact differently with the impurities. The general expansion around the TRSB state is:

$$\begin{aligned}\Delta_{h_1} &= \Delta e^{i\alpha}(1 + m_1 + i\phi_1) \\ \Delta_{h_2} &= \Delta e^{-i\alpha}(1 + m_2 + i\phi_2) \\ \Delta_{e_1} &= -\eta\Delta(1 + m_{e_1} + i\phi_{e_1}) \\ \Delta_{e_2} &= -\eta\Delta(1 + m_{e_2} + i\phi_{e_2}).\end{aligned}\tag{30}$$

Case(a): equivalent gap fluctuations on hole pockets and inequivalent gap fluctuations on the electron pockets: To realize this case, we take $a_{h_1e_1} \neq a_{h_1e_2}$, but $a_{h_1e_i} = a_{h_2e_i}$. This case is realized when we include into consideration the hybridization between the two elliptical electron pockets, which gives rise to the splitting of the two electron pockets in the folded Brillouin zone into non-equivalent inner and outer pockets, each of which respects C_4 symmetry. Note from Eq. (11) in the MT that $a_{1,2,3,4}$ do not really affect qualitatively our results as they always enter in combination with a_h or a_e ; thus we may even set them to zero without qualitatively affecting the result. With this assumption, the current is:

$$\begin{aligned}j_x &= j_0 [(a_h - (a_{he_1} + a_{he_2})\eta \cos \alpha) \partial_x (\phi_1 + \phi_2) + (a_h \sin 2\alpha + (a_{he_1} + a_{he_2})\eta \sin \alpha) \partial_x (m_1 - m_2) \\ &\quad + (a_e \eta^2 - 2a_{he_1} \eta \cos \alpha) \partial_x \phi_{e_1} + (a_e \eta^2 - 2a_{he_2} \eta \cos \alpha) \partial_x \phi_{e_2}],\end{aligned}\tag{31}$$

and $j_y = (x \rightarrow y)$. The GL equations are:

$$\begin{pmatrix} 0 & a_h & -a_{h_1e_1} & -a_{h_1e_2} \\ a_h & 0 & -a_{h_2e_1} & -a_{h_2e_2} \\ -a_{h_1e_1} & -a_{h_2e_1} & 0 & a_e \eta \\ -a_{h_1e_2} & -a_{h_2e_2} & a_e \eta & 0 \end{pmatrix} \begin{pmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \end{pmatrix} = -R \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix},\tag{32}$$

where $g_1 = e^{i\alpha}(m_1 + i\phi_1)$, $g_2 = e^{-i\alpha}(m_2 + i\phi_2)$, $g_3 = \eta(m_{e_1} + i\phi_{e_1})$, $g_4 = \eta(m_{e_2} + i\phi_{e_2})$; and

$$R \equiv \int \frac{d^2k}{(2\pi)^2} e^{i\vec{k} \cdot \vec{r}} \frac{\tilde{\alpha}_{\text{imp}}}{k^2 + \delta^2}.\tag{33}$$

After matrix inversion, we get $\phi_{e_1, e_2} = 0$ (by looking at the imaginary part of the solutions for g_i). Since $a_{h_1e_i} = a_{h_2e_i}$ we find $m_1 = m_2$ and $\phi_1 = -\phi_2$. This leads to the vanishing of the supercurrent.

Case(b) inequivalent gap fluctuations of the hole pockets and equivalent gap fluctuations on the electron pockets: In this case, we treat electron pockets as circular and neglect the hybridization between them, but no longer assume that fluctuations on the two hole pockets are identical. Since we are probing here the effect of inequivalent fluctuations of the hole pockets, the term a_h , which was already included in previous analysis, may be safely set to zero. The same argument allows us to set $a_e = 0$. In distinction with the previous case, we, however, need to keep a_1 and a_2 . We found in the MT that $a_1 + a_2$ does not contribute to the current. We use this and set from the beginning $a_1 = \tilde{\varepsilon}$ and

$a_2 = -\tilde{\epsilon}$. We keep $a_{h_1e_1} = a_{h_1e_2}$, but set $a_{h_1e_1} \neq a_{h_2e_1}$. The supercurrent is given by (using the definition given in the MT)

$$\begin{aligned} j_x &= j_0 \left[\tilde{\epsilon} \partial_x (\phi_1 - \phi_2) + 2a_{h_1e} \left\{ -\cos \alpha \partial_x \left(\phi_1 + \frac{\tilde{\phi}_e}{2} \right) - \sin \alpha \partial_x \left(m_1 - \frac{\tilde{m}_e}{2} \right) \right\} \right. \\ &\quad \left. + 2a_{h_2e} \left\{ -\cos \alpha \partial_x \left(\phi_2 + \frac{\tilde{\phi}_e}{2} \right) + \sin \alpha \partial_x \left(m_2 - \frac{\tilde{s}_e}{2} \right) \right\} \right], \\ j_y &= (x \rightarrow y), \end{aligned} \quad (34)$$

where $\tilde{\phi}_e = \phi_{e_1} + \phi_{e_2}$ and $\tilde{m}_e = m_{e_1} + m_{e_2}$. The GL equations now read:

$$\begin{pmatrix} \tilde{\epsilon} & 0 & -a_{h_1e} \\ 0 & -\tilde{\epsilon} & -a_{h_2e} \\ -a_{h_1e} - a_{h_2e} & -a_{h_1e} - a_{h_2e} & 0 \end{pmatrix} \begin{pmatrix} g_1 \\ g_2 \\ \tilde{g}_3 \end{pmatrix} = -R \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad (35)$$

where $\tilde{g}_e = g_{e_1} + g_{e_2}$. The solution to the GL equations are:

$$m_1 + i\phi_1 = -R \frac{a_{h_1e}^2 - a_{h_2e}^2 - a_{h_1e}\tilde{\epsilon}}{\tilde{\epsilon}(a_{h_1e}^2 - a_{h_2e}^2)} e^{-i\alpha}, \quad (36)$$

$$m_2 + i\phi_2 = R \frac{a_{h_1e}^2 - a_{h_2e}^2 - a_{h_2e}\tilde{\epsilon}}{\tilde{\epsilon}(a_{h_1e}^2 - a_{h_2e}^2)} e^{i\alpha}, \quad (37)$$

$$\tilde{g}_e = R \frac{\tilde{\epsilon}}{a_{h_1e}^2 - a_{h_2e}^2}, \quad (38)$$

$$\tilde{\phi}_e = 0. \quad (39)$$

Plugging this back into Eq. (34) we see that the $\sin \alpha$ and $\cos \alpha$ terms of $m_{1,2}$ and $\phi_{1,2}$ cancel each other out, and the variations of fluctuations of the gap amplitude \tilde{m}_e cancels the variation of the relative phase $\phi_1 - \phi_2$. Thus there is no supercurrent.

Case(c) inequivalent gap fluctuations on the two hole pockets and on the two electron pockets: This case is a combination of cases (a) and (b) and it is realized if we set $a_{h_1e_1} \neq a_{h_1e_2}$. The current is then given by:

$$\begin{aligned} j_x &= j_0 [\tilde{\epsilon} \partial_x (\phi_1 - \phi_2) - \cos \alpha \{ (a_{h_1e_1} + a_{h_1e_2}) \partial_x \phi_1 + (a_{h_2e_1} + a_{h_2e_2}) \partial_x \phi_2 \} \\ &\quad - \sin \alpha \{ (a_{h_1e_1} + a_{h_1e_2}) \partial_x m_1 - (a_{h_2e_1} + a_{h_2e_2}) \partial_x m_2 \} \\ &\quad + \sin \alpha \{ (a_{h_1e_1} - a_{h_2e_1}) \partial_x m_{e_1} + (a_{h_1e_2} - a_{h_2e_2}) \partial_x m_{e_2} \}] \\ j_y &= (x \rightarrow y). \end{aligned} \quad (40)$$

The GL equations are given by:

$$\begin{pmatrix} \tilde{\epsilon} & 0 & -a_{h_1e_1} & -a_{h_1e_2} \\ 0 & -\tilde{\epsilon} & -a_{h_2e_1} & -a_{h_2e_2} \\ -a_{h_1e_1} & -a_{h_2e_1} & 0 & 0 \\ -a_{h_1e_2} & -a_{h_2e_2} & 0 & 0 \end{pmatrix} \begin{pmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \end{pmatrix} = -R \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad (41)$$

The solutions are given by:

$$m_1 + i\phi_1 = -R \frac{a_{h_2e_1} - a_{h_2e_2}}{a_{h_1e_1}a_{h_2e_2} - a_{h_2e_1}a_{h_1e_2}} e^{-i\alpha}, \quad (42)$$

$$m_2 + i\phi_2 = R \frac{a_{h_1e_1} - a_{h_1e_2}}{a_{h_1e_1}a_{h_2e_2} - a_{h_2e_1}a_{h_1e_2}} e^{i\alpha}, \quad (43)$$

$$\eta m_{e_1} = -R \frac{a_{h_1e_2}a_{h_2e_2}(a_{h_2e_1} + a_{h_1e_1}) - a_{h_1e_2}^2a_{h_2e_1} - a_{h_2e_1}^2a_{h_1e_2} + (-a_{h_1e_1}a_{h_1e_2} + a_{h_1e_2}^2 - a_{h_2e_2}^2 + a_{h_2e_1}a_{h_2e_2})\tilde{\epsilon}}{(a_{h_1e_1}a_{h_2e_2} - a_{h_2e_1}a_{h_1e_2})^2}, \quad (44)$$

$$\eta m_{e_2} = -R \frac{a_{h_1e_1}a_{h_2e_1}(a_{h_2e_2} + a_{h_1e_2}) - a_{h_1e_1}^2a_{h_2e_2} - a_{h_2e_1}^2a_{h_1e_2} + (-a_{h_1e_1}a_{h_1e_2} + a_{h_1e_1}^2 - a_{h_2e_1}^2 + a_{h_2e_1}a_{h_2e_2})\tilde{\epsilon}}{(a_{h_1e_1}a_{h_2e_2} - a_{h_2e_1}a_{h_1e_2})^2}, \quad (45)$$

$$\phi_{e_1} = 0, \quad (46)$$

$$\phi_{e_2} = 0. \quad (47)$$

Plugging this back into Eq. (40), we once again find that the $\sin \alpha$ and $\cos \alpha$ terms of $m_{1,2}$ and $\phi_{1,2}$ cancel each other out, and the variations of the gap magnitudes m_3 and m_4 cancel the variation of the relative phase $\phi_1 - \phi_2$, leading to vanishing of the supercurrent.

Thus we have seen that neither the in-equivalence of gap fluctuations on the hole pockets nor that on the electron pockets induces supercurrents in the $s + is$ state as long as C_4 symmetry is maintained.

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